Comment on "Dynamic properties in a family of competitive growing models"

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The article [Phys. Rev. E **73**, 031111 (2006)] by Horowitz and Albano reports on simulations of competitive surface-growth models RD+X that combine random deposition (RD) with another deposition X that occurs with probability p. The claim is made that at saturation the surface width $w(p)$ obeys a power-law scaling $w(p) \propto 1/p^{\delta}$, where δ is only either $\delta = 1/2$ or $\delta = 1$, which is illustrated by the models where X is ballistic deposition and where X is RD with surface relaxation. Another claim is that in the limit $p \to 0^+$, for any lattice size L, the time evolution of $w(t)$ generally obeys the scaling $w(p,t) \propto (L^{\alpha}/p^{\delta})F(p^{2\delta}t/L^{z})$, where F is Family-Vicsek universal scaling function. We show that these claims are incorrect.

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In Ref.[1] the following scaling ansatz is proposed:

$$w^2(p,t) \propto \frac{L^{2\alpha}}{p^{2\delta}} F\left(p^{2\delta}\frac{t}{L^z}\right), \qquad (1)$$

where $w(p,t)$ are time evolutions of surface width in competitive growth models RD+X when a random deposition (RD) process is combined with process X, and $p \in (0;1]$ is the selection probability of process X. The function $F(\cdot)$ represents Family-Vicsek universal scaling. The anszatz (1) has been studied previously [2, 3, 4] by examples where X represented either Kardar-Parisi-Zhang or Edwards-Wilkinson universal process. The new claim that is being made in Ref.[1] is that a nonuniversal and *model-dependent* exponent δ in Eq.(1) must be only of two values, either $\delta = 1$ or $\delta = 1/2$, for models studied in Ref.[1]. To show that this claim is not correct we performed $(1+1)$ dimensional simulations of RD+X models when X is ballistic deposition (BD) and when X is random deposition with surface relaxation (RDSR), and performed scaling in accordance to Ref.[1]. Our results are presented in figs.1-3.

Our data have been obtained on L site lattices (L is indicated in the figures) with periodic condition, starting from initially flat substrates, and averaged over 400 to 600 independent configurations. The time t is measured in terms of the deposited monolayers. Simulations have been carried up to $t = 10^7$, and the surface width at saturation has been averaged over the last 5000 time steps. The data sets are for ten equally spaced selection probabilities p from $p = 0.1$ to $p = 1$, where $p = 0$ would be for RD process with no X present, and $p = 1$ is for process X in the absence of RD. The data have been scaled in L with the theoretical values of universal roughness exponent α and dynamic exponent z of the universality class of process X. The RDSR algorithm used in our simulations is given in Ref.[5] (Sec.5.1). The BD algorithm used as X=BD1 is the nearest-neighbor (NN) sticking rule found in Ref.[5] (Sec.2.2), and the BD algorithm used as X=BD2 is the next-nearest-neighbor (NNN) sticking rule found in Ref.[5] (Sec.8.1).

Saturation. Saturation data (fig.1) show that in special cases an approximate power law $w(p) \propto 1/p^{\delta}$ may be observed. However, this is not a principle. Even if the data can be fit to the power law in p only one of our examples shows a reasonable fit with $\delta \approx 1$ (seen in fig.1a). When X=BD1 the data in fig.1b show $\delta < 1/2$. The other two examples shown in fig.1 defy a linear fit.

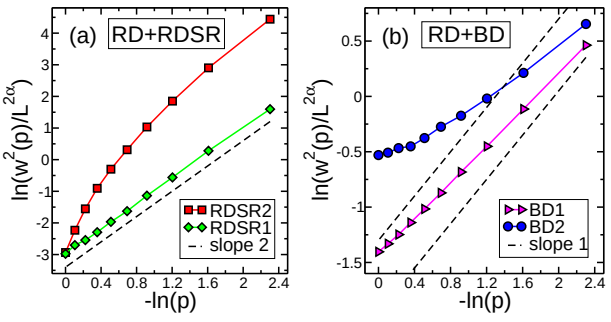

FIG. 1: (color on line) Interface width at saturation in the RD+X model vs the selection probability p of process X. (a) X is RDSR: the case when both RD and RDSR deposits are of unit height (diamonds, RDSR1; $L = 500$); and, the case when RDSR deposits are of unit height and RD deposits are of twice that height (squares, RDSR2; $L = 100$). (b) X is BD: the case of the NNN rule (circles, BD2); and, the case of the NN rule (triangles, BD1). In RD+BD simulations $L = 500$. Solid line segments connecting data points (symbols) are guides for the eye. The dashed lines give reference slopes.

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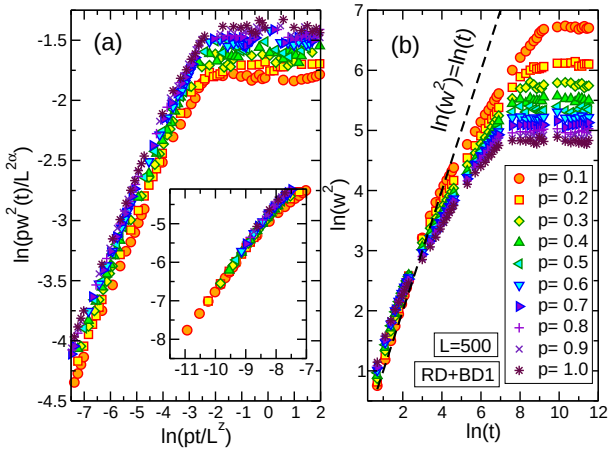

FIG. 2: (color on line) Time-evolutions $w^2(p,t)$ in RD+BD1. (a) Scaling in p after Ref.[1]. The insert shows the scaled initial transients. (b) Evolution curves before scaling. The dashed line is the RD evolution for $p = 0$. In all models when the simulations start from a flat substrate $w(t)$ must pass an initial transient before universal scaling can be measured. The initial transients in part (b) follow RD universal evolution.

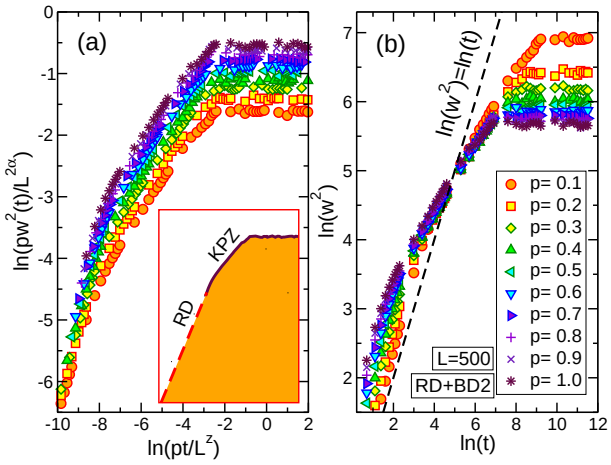

FIG. 3: (color on line) Time-evolutions $w^2(p,t)$ in RD+BD2. (a) Scaling in p after Ref.[1]. The outcome of this scaling is summarized in the insert. (b) Data before scaling. The dashed line is the RD evolution for $p = 0$.

In these cases there is no power law of the type claimed in Ref.[1]. This absence of power-law scaling in p is also evident in fig.4 of Ref.[1].

The RD limit. Another claim of Ref.[1] is that Eq.(1) with the power-law prefactors p^δ (where $\delta = 1$ or $1/2$) would prevail in the RD limit of $p \to 0^+$, and that such a scaling would be universal. We tested these claims in simulations of RD+BD models and found the evidence to the contrary (figs.2-3). In order to prove the absence of power-law scaling via Eq.(1) in the RD limit we present in figs.2b-3b the original $w^2(p,t)$ data before scaling. These original data show that parameter p, $p \in (0;1]$, assigns an order in the set of all curves $w^2(p,t)$ in such a way that $w^2(1,t)$ is the lowest lying curve, and at $p = 0$ the initial transients become the RD universal evolution $w^2_{RD}(0,t) \propto t$. The region between the boundaries $w^2(1,t)$ and $w^2_{RD}(0,t)$ is densely covered by the curves $w^2(p,t)$ because p takes on continuous values. The pattern shown in figs.2b-3b for $p \in [0.1;1]$ extends down to values that are infinitesimally close to $p = 0$, i.e., to the entire range of p. If the simulations are stopped at infinitesimally small p' the width $w^2(p',t)$ is always the highest lying curve in figs.2b-3b. In other words, the smaller the p' the higher the saturation value of $w^2(p',t)$. But there is no bounding highest curve $w^2(p',t)$ in this set since the boundary $w^2(0,t)$ is the RD evolution. This order is reversed under the scaling of Eq.(1) when we set $\delta = 1/2$, following Ref.[1]. The outcome of this scaling is seen in figs.2a-3a: the boundary $w^2(1,t)$, i.e., the lowest-lying curve in figs.2b-3b, is mapped onto the highest-lying curve in the image of this scaling seen in figs.2a-3a; and, a higher-lying curve $w^2(p,t)$ before scaling in figs.2b-3b is mapped onto a lower-lying curve after scaling in figs.2a-3a. In this scaling, the initial transients become ever longer as p becomes ever smaller and closer to $p = 0$, as seen in the insert in fig.2a. For any range of p, also in the limit $p \to 0^+$, the image of this scaling demonstrates no data collapse. This image is shown in the insert in fig.3a. Hence, for RD+BD models Eq.(1) with $\delta = 1/2$ does not produce data collapse.

In some instances of model X, however, Eq.(1) can give an *approximate* data collapse [3, 4] but then δ is not restricted to the two values postulated in Ref.[1]. For example, for the RD+BD1 model such scaling can be obtained with $\delta \approx 0.41$ [3] (note, $0.4 < \delta < 0.5$ is seen in fig.1b). But for the RD+BD2 model there is no value of δ that would produce data collapse when nonuniversal prefactors in Eq.(1) are expressed as a power law p^δ. We have demonstrated that such scaling does not generally exist and if occasionally it is observed it is a property of particular model.

In summary, the form of the nonuniversal prefactors as seen in universal Eq.(1) is a fit and is *not* a principle. The exponent δ in Eq.(1) is model dependent, and the prefactor that enters may have other forms than p^δ.

[1] C. M. Horowitz and E. V. Albano, Phys. Rev. E, **73** 031111 (2006).
[2] A. Kolakowska *et al* , Phys. Rev. E **70**, 051602 (2004).
[3] A. Kolakowska *et al* , Phys. Rev. E **73**, 011603 (2006).
[4] A. Kolakowska and M. A. Novotny, cond-mat/0511688.
[5] A.-L. Barabasi and H. E. Stanley, *Fractal Concepts in Surface Growth* (Cambridge University Press, Cambridge, 1995).